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# Nonlinear Control Design for Linear Inverted Pendulum System using Exact Feedback Linearization

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Exact feedback linearization converts any system into appropriate form such that the controller design becomes convenient for the system. In this technique, control algorithm is developed based on exact feedback linearization through energy control. This approach is applicable to under actuated in which single control input, acceleration of cart, controlled the angle and position of cart. This technique provides simple solution based on Exact Feedback Linearization but saturation of control signal which causes problem in controller design of LIP which can be alleviated by introduction of mechanism of angle reference switching.

**Keywords :** Energy Control, Exact Feedback Linearization, Reference Switching.

## 1. INTRODUCTION

Among all Nonlinear Control system, Linear Inverted Pendulum (LIP) [1] poses special nonlinear control characteristics. Nonlinear property of Linear Inverted Pendulum demonstrate dynamics of many control system such as Rocket Launch pad and single link Robot manipulator which are having analogous mathematical model system. Linear inverted pendulum basically highly nonlinear under actuated system having unstable equilibrium point at upright vertical position. LIP control problem is categorized into two section (1) Swing up (2) Stabilization. Many advanced techniques are employed such as neural, fuzzy PD, LQR and Backstepping over conventional techniques to make system swing up from its pendant position and stabilize at its upright equilibrium position.

Nonlinear control design mostly based on Lyapunov stability theory such as Sliding mode control, Adaptive control *etc.*, but there are some techniques in which design is not based on Lyapunov stability theory such as Exact Feedback Linearization. Feedback Linearization is a control technique which is prominently used for Nonlinear System. Feedback linearization [2-14] Concept is based on transforming nonlinear system into Linear system which is introduced by Isidori in 1989. In Feedback Linearization, Nonlinearity is converted into linear through the help of Axes Transformation and cancellation of Nonlinearity with help of control signal so that tools of Linear control philosophy is applicable to transformed system without making any approximation of nonlinear model into linear model. This approach provides powerful tool which converts a very complex nonlinear system into linear system which is very easy to control.

In this paper, Exact Feedback Linearization control technique is applied on LIP for the swing up of pendulum from its pendant position to upright equilibrium position through mechanism of energy control and angle reference. Angle reference pumps energy into system when LIP system does not have sufficient energy in order to place pendulum of LIP at upright equilibrium position from pendant position.

## 2. EXACT FEEDBACK LINEARIZATION FOR SWING UP OF LIP

The mathematical model of the LIP [1] is given by equilibrium point with its constrained control signal (-1.5 to 1.5) because angle reference provides extra pumping of pendulum in order to arrive at its upright equilibrium position.

Figure 5 shows response of angle of LIP and corresponding control signal in absence of switching strategy of angle reference in which angle is stabilized at its upright equilibrium point with its unconstrained control signal (-$\infty$ to $\infty$ ) but control effort given by controller increased very large as compared to control signal in presence of switching of angle reference and hence switching strategy of angle reference provides optimal control strategy to control LIP with less control effort.

Figure 6 and Figure 7 shows response of switching of angle and reference angle of pendulum for different restricted control signal (u1 and u2) in which switching is occurred whenever angular velocity becomes zero and switching frequency is increased as strength of control signal is reduced.Figure 8 and Figure 9 show the response of angle and position and corresponding control effort in presence of noise power of strength 0.5 in which response of angle and position became unstable. This unstable response is happened due to exact cancelation of terms cannot be done in presence of noise which makes system unstable.

## 4. CONCLUSIONS

In Exact Feedback Linearization control algorithm, a global stabilization (GSA) of LIP has been achieved as shown in result but there are problems such as singularity and unstable zero dynamics in the design of LIP. The singularity problem is rectified through help of constrained control signal, GSA of LIP is achieved through switching of angle reference. In specific, introduction of concept of angle reference provide a great promise to deal with saturation and LIP. But control technique is based on exact cancellation of nonlinearity present in system which causes complete information of mathematical model of system to implement exact feedback linearization control technique.

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